# NATIONAL BUREAU OF STANDARDS REPORT



## CHECK OF METHOD FOR COMPUTING INFLUENCE COEFFICIENTS OF DELTA AND OTHER WINGS

by

Ruth M. Woolley

Report to
Bureau of Aeronautics
Department of the Navy

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

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To
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NBS

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# CHECK OF METHOD FOR COMPUTING INFLUENCE COEFFICIENTS OF DELTA AND OTHER WINGS


## SUMMARY

Computations, based on a proposed method of computing the influence coefficients of a delta wing, were made to determine the influence coefficients of a swept-back model wing for which experimental results were available. Comparison of computed results with experimental data showed poor agreement. The lack of agreement is in part attributed to insufficient knowledge regarding the shear stiffness of ribs and bulkheads.


## INTRODUCTION

With the continuing demand for increased speed in aircraft, there has been a change in wing shape from the familiar thick, straight wing to the thin swept-back wing and to the delta wing. For the swept and delta wings conventional methods of analysis using beam theory do not give satisfactory results. In the past few years newer types of analysis based on energy methods, on plate theory, on continuity considerations, and on the assumption of constant chordwise slope of multi-spar wings have been developed. (Ref. 1) These methods were generally satisfactory for swept-back wings but for delta wings and for wing-fold joints in swept wings, there was not sufficiently close agreement of computed values with experimental ones.

The method of reference 1, used for the computations for this report, combines the interaction of ribs, spars and torque cells. It was intended to give improved accuracy and to take account of secondary effects, such as chordwise bending, discontinuities in the cover sheets and the actual support condition at the root of the individual spars. The load carried by the wing was considered to be the sum of (1) that carried by spars in bending, (2) that carried by the ribs in bending, and (3) that carried by the cover sheet in torsion. The load carried by each of these three mechanisms was determined in terms of the deflections at the junctions of ribs and spars. The sum of these loads expressed in terms of deflections, was equated to the applied external forces at the junctions. The equations which result were solved for the deflections at the junctions. From the deflections, the

bending and torsional stresses were in turn determined, thus giving a complete deformation and stress analysis of the wing.

For this report the method of analysis described above was applied to a swept-back wing model which had already been tested at the National Bureau of Standards. The computed results were then compared with the experimental data.

## APPLICATION

In this computation, the swept-back wing (figure 1) was considered to be divided into three spanwise beams: - a main spar, a leading edge spar and a trailing edge spar. These beams were separated by lines, CD and EF, figure 2, midway between the center lines of the adjacent spars.

The wing was also considered to be divided into four chordwise beams shown in figure 3 and a root beam which does enter in the computation. The effective width of the sheet to each side of the rib was taken to be 0.181 of the total rib length (see fig. 3) as recommended in reference 1. Only the cover sheet and ribs were considered as contributing to the bending stiffness of the chordwise beams. The values of EI computed for a number of stations along the spars and ribs are given in table 1.

The next step was to find the clamped root influence coefficients for the spars and ribs, following the method described in reference 2. The necessary scale factors (reference 2, pp. 5-6), the EI, the distances between stations and the number of stations were tabulated and read into SEAC together with the code for computation of influence coefficients for tapered cantilever beams in bending. The results gave, for each beam, a series of equations of deflections in terms of loads. The results are given in table 2. For example, the first line of table 2 written as an equation reads:

$$y_1 = 10^{-6}\left[16.791301L_1 + 43.716262L_4 + 70.641223L_7 + 96.594665L_{10}\right]$$

where

$y_1$ is the deflection at station 1
$L_1$ is the load at station 1
$L_4$ is the load at station 4, etc.
the subscripts are stations in figure 4

For the main spar, where the root was clamped, inverting the influence coefficient matrix was sufficient to give the loads in terms of deflections, according to equation (2b) of reference (1):

$$\{L\} = [\delta]^{-1} \{y\}$$

(clamped root condition)

For the other two spars, the beams were simply supported at the root, and the loads in terms of deflections were computed using equation (10) of reference (1):

$$\{L\} = \left[ [\delta]^{-1} - \frac{[\delta]^{-1} \{\ell\} [\ell] [\delta]^{-1}}{[\ell] [\delta]^{-1} \{\ell\}} \right] \{y\}$$

(simply-supported root condition)

The ribs were considered free at the root, and for them equations (11) and (12) of reference (1) were used:

$$\{L\} = \left[ [\delta]^{-1} - \frac{[\delta]^{-1} \{\ell\} [\ell] [\delta]^{-1}}{[\ell] [\delta]^{-1} \{\ell\}} \right] \{y - y_t\}$$

(free root condition)

for loads at stations other than the root station t and at root station t,

$$L_t = -L_r - L_s - \ldots L_j - \ldots - L_n$$

(free root condition)

Table 3 gives the matrix of loads in terms of deflections for each of the spanwise and chordwise beams with their appropriate root fixity.

The torsional stiffness, GJ, was computed for each of the eight torque boxes of the wing according to the method described in reference (1), paragraph 1 of page 5. A torque box is that formed from two adjacent ribs, two adjacent spars, and the cover sheets. Using an X-X axis midway between the spars as the torque axis, the GJ for the cross-section normal to the X-X axis at its mid-point was computed as representative of the box, assuming for convenience the sheet thickness of the spars to be infinite. The cross-section for computation of GJ for a leading edge torque box is given in figure 5. The effective GJ was computed as

$$GJ = \frac{4 A^2 G}{\int_0 \frac{ds}{t}}$$

where (see figure 5)

s is the midline of the torque box. In the spar s is located at the midthickness. In the cover sheets, where the corrugation and outer surface are riveted together, s is located at the midthickness of the corrugation plus outer surface. In the cover sheets, where the corrugation stands out, s is located between the corrugation and outer skin at a distance from them inversely proportional to their respective thicknesses.

A is the area enclosed by s.

t is the thickness of the walls of the torque box. In the cover sheets, thickness is taken as outer sheet thickness plus corrugation thickness. In the spar, the walls are considered thick enough to be taken as infinite.

The GJ values obtained are given in table 4. The loads were computed in terms of deflections according to equation (18) of reference (1). These results are given in table 5.

When all the loads had been found in terms of deflections due to torque and to spanwise and chordwise bending, the total load P was found according to equations (19) and (20) of reference (1):

$$\begin{aligned} P_r &= \Sigma L_r \\ P_s &= \Sigma L_s \\ &\vdots \\ P_j &= \Sigma L_j \\ &\vdots \\ P_n &= \Sigma L_n \end{aligned}$$

which can be written as $\{P\} = [\Delta]\{y\}$

This matrix is given in table 6.

The deflections in terms of load were found by inverting the above matrix, as indicated in equation (21) of reference (1).

$$\{y\} = [\Delta]^{-1} \{p\}$$

The value of $[\Delta]^{-1}$ is given in table 7.

## RESULTS

The results given in table 7 were compared with available experimental data, table 8. There was poor agreement, the computed values varying from 63 percent less (at 3, 3) to 5 percent more (at 1, 12) than experimental values.

Because of the poor agreement, particularly in the root region, consideration was given to possible causes for the discrepancy. A thorough check of the experimental techniques and computational procedure indicated that both were done accurately and that such large differences could not be the result of error. The only possible cause for the difference seemed to lie in the fact that the computations assumed the bulkheads rigidly connected to the cover sheets; whereas, actually they were riveted by single rivets to the tops of the corrugations. This could have made the structure much weaker in carrying torsional loads. To approximate this in the computation all the torsional stiffness matrices, table 5, were modified arbitrarily. The torsional stiffness of cells I and V, figure 4, was taken as zero, that of cells II and VI as 1/10 their original value, and that of the remaining cells as their original values. With these values the computation was repeated. The resulting load matrix is given in table 9 and its inverse, the influence coefficient matrix is given in table 10.

Comparison of tables 7, 8, and 10 indicates some improvement due to considering the shear stiffness of the bulkheads to be reduced. The discrepancy still remaining, particularly at (12,12) may also be due to using too high a shear stiffness, in this case for the spars. The computations neglect shear deformation of the spars, while for the heavy, thin, riveted spars used these deformations may not be negligible.

## CONCLUSIONS

The lack of agreement of computed and experimental values is considered to be primarily due to lack of knowledge regarding the shearing deformations. Experiments on elementary structures are needed to provide better information in this regard.

For the Director,

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## REFERENCES

(1) Levy, Samuel: Structural Analysis and Influence Coefficients for Delta Wings, NBS Report No. 2031 for Bureau of Aeronautics, Dept. of Navy, Oct. 1952.

(2) Levy, Samuel: Influence Coefficients of Tapered Beams Computed on SEAC, NBS Report No. 1464 for Bureau of Aeronautics, Dept. of Navy, Feb. 1952.

Table 1-Bending stiffness of equivalent spar and rib beams for computing influence coefficients

$E = 10.5\ (10)^6$ lbs/in.$^2$
$\ell_m$ = distance from station m-1 to station m
$I_m$ = moment of inertia of a cross-section at station m

| | m | $\ell_m$, in. | $10^{-6}EI_m$, lb. in.$^2$ |
|---|---|---|---|
| leading edge spar | 0 | | 24.286 |
| | 1 | 2.27 | 28.886 |
| | 2 | 8.74 | 21.976 |
| | 3 | 11.64 | 13.398 |
| | 4 | 11.64 | 7.754 |
| | 5 | 11.22 | 1.064 |
| main spar | 0 | | 69.142 |
| | 1 | 3.17 | 74.361 |
| | 2 | 7.23 | 52.500 |
| | 3 | 11.00 | 30.555 |
| | 4 | 11.00 | 15.918 |
| | 5 | 10.60 | 4.838 |
| trailing edge spar | 0 | | 37.306 |
| | 1 | 9.83 | 27.069 |
| | 2 | 10.39 | 15.498 |
| | 3 | 10.39 | 9.825 |
| | 4 | 6.37 | 7.113 |
| | 5 | 3.65 | 3.567 |
| rib 2 | 0 | | 5.734 |
| | 1 | 5.945 | 5.734 |
| | 2 | 9.693 | 5.734 |
| rib 3 | 0 | | 2.948 |
| | 1 | 4.883 | 2.948 |
| | 2 | 8.567 | 2.948 |
| rib 4 | 0 | | 1.307 |
| | 1 | 3.834 | 1.307 |
| | 2 | 7.432 | 1.307 |
| rib 5 | 0 | 0 | 0.4095 |
| | 1 | 2.808 | 0.4095 |
| | 2 | 6.347 | 0.4095 |

Table 2-Matrix $[\delta]$ for computing deflections in terms of loads, for spars and ribs considered as cantilever beams, $d_j = [\delta]\{L\}$.
Subscripts refer to stations in figure 4.

leading edge spar

$$\begin{bmatrix} d_1 \\ d_4 \\ d_7 \\ d_{10} \end{bmatrix} = 10^{-6} \begin{bmatrix} 16.791301 & 43.716262 & 70.641223 & 96.594665 \\ 43.716262 & 157.487488 & 287.048389 & 411.934412 \\ 70.641223 & 287.048389 & 616.820283 & 960.486274 \\ 96.594665 & 411.934412 & 960.486274 & 1752.424891 \end{bmatrix} \begin{bmatrix} L_1 \\ L_4 \\ L_7 \\ L_{10} \end{bmatrix}$$

main spar

$$\begin{bmatrix} d_2 \\ d_5 \\ d_8 \\ d_{11} \end{bmatrix} = 10^{-6} \begin{bmatrix} 5.360072 & 14.061974 & 22.763875 & 31.149344 \\ 14.061974 & 52.298946 & 96.278684 & 138.659159 \\ 22.763875 & 96.278684 & 212.140098 & 334.001058 \\ 31.149344 & 138.659159 & 334.001058 & 604.296267 \end{bmatrix} \begin{bmatrix} L_2 \\ L_5 \\ L_8 \\ L_{11} \end{bmatrix}$$

trailing edge spar

$$\begin{bmatrix} d_3 \\ d_6 \\ d_9 \\ d_{12} \end{bmatrix} = 10^{-6} \begin{bmatrix} 9.289544 & 24.441738 & 39.593932 & 54.206539 \\ 24.441738 & 89.807517 & 164.657277 & 236.841549 \\ 39.593932 & 164.657277 & 357.840868 & 559.135760 \\ 54.206539 & 236.841549 & 559.135760 & 971.277820 \end{bmatrix} \begin{bmatrix} L_3 \\ L_6 \\ L_9 \\ L_{12} \end{bmatrix}$$

rib 2

$$\begin{bmatrix} d_2 \\ d_3 \end{bmatrix} = 10^{-6} \begin{bmatrix} 12.214526 & 42.087208 \\ 42.087208 & 222.312901 \end{bmatrix} \begin{bmatrix} L_2 \\ L_3 \end{bmatrix}$$

rib 3

$$\begin{bmatrix} d_5 \\ d_6 \end{bmatrix} = 10^{-6} \begin{bmatrix} 13.164714 & 47.810048 \\ 47.810048 & 275.117426 \end{bmatrix} \begin{bmatrix} L_5 \\ L_6 \end{bmatrix}$$

rib 4

$$\begin{bmatrix} d_8 \\ d_9 \end{bmatrix} = 10^{-6} \begin{bmatrix} 14.373399 & 56.166478 \\ 56.166478 & 364.680368 \end{bmatrix} \begin{bmatrix} L_8 \\ L_9 \end{bmatrix}$$

rib 5

$$\begin{bmatrix} d_{11} \\ d_{12} \end{bmatrix} = 10^{-6} \begin{bmatrix} 18.022546 & 79.127838 \\ 79.127838 & 624.597000 \end{bmatrix} \begin{bmatrix} L_{11} \\ L_{12} \end{bmatrix}$$

Table 3- Matrix $[\delta]^{-1}$ for computing loads in terms of deflections for spars and ribs, taking account of root condition:

$$\{L\} = [\delta]^{-1}\{y\}$$

Subscripts refer to stations in figure 4.

leading edge spar

$$\begin{bmatrix} L_1 \\ L_4 \\ L_7 \\ L_{10} \end{bmatrix} = 10^6 \begin{bmatrix} .124734 & -.0976097 & .0280772 & -.00275172 \\ -.0976097 & .105965 & -.0500207 & .00856489 \\ .0280772 & -.0500207 & .0390029 & -.0112848 \\ -.00275172 & .00856489 & -.0112848 & .00490565 \end{bmatrix} \begin{bmatrix} y_1 \\ y_4 \\ y_7 \\ y_{10} \end{bmatrix}$$

main spar

$$\begin{bmatrix} L_2 \\ L_5 \\ L_8 \\ L_{11} \end{bmatrix} = 10^6 \begin{bmatrix} .777932 & -.355439 & .0968070 & -.0120485 \\ -.355439 & .316882 & -.154424 & .0309630 \\ .0968070 & -.154424 & .126966 & -.039732\underline{4} \\ -.0120485 & .0309630 & -.039732\underline{4} & .0171318 \end{bmatrix} \begin{bmatrix} y_2 \\ y_5 \\ y_8 \\ y_{11} \end{bmatrix}$$

trailing edge spar

$$\begin{bmatrix} L_3 \\ L_6 \\ L_9 \\ L_{12} \end{bmatrix} = 10^6 \begin{bmatrix} .214208 & -.171456 & .0553350 & -.00818668 \\ -.171456 & .197616 & -.109643 & .0257392 \\ .0553350 & -.109643 & .10003\underline{4} & -.0341868 \\ -.00818668 & .0257392 & -.0341868 & .0149271 \end{bmatrix} \begin{bmatrix} y_3 \\ y_6 \\ y_9 \\ y_{12} \end{bmatrix}$$

rib 2

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = 10^6 \begin{bmatrix} .0311240 & -.0502133 & .0190893 \\ -.0502133 & .0810106 & -.0307973 \\ .0190893 & -.0307973 & .0117080 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

rib 3

$$\begin{bmatrix} L_4 \\ L_5 \\ L_6 \end{bmatrix} = 10^6 \begin{bmatrix} .0275698 & -.0432858 & .0157160 \\ -.0432858 & .0679606 & -.0246748 \\ .0157160 & -.0246748 & .00895882 \end{bmatrix} \begin{bmatrix} y_4 \\ y_5 \\ y_6 \end{bmatrix}$$

rib 4

$$\begin{bmatrix} L_7 \\ L_8 \\ L_9 \end{bmatrix} = 10^6 \begin{bmatrix} .0236767 & -.0358910 & .0122143 \\ -.0358910 & .0544064 & -.0185154 \\ .0122143 & -.0185154 & .00630109 \end{bmatrix} \begin{bmatrix} y_7 \\ y_8 \\ y_9 \end{bmatrix}$$

rib 5

$$\begin{bmatrix} L_{10} \\ L_{11} \\ L_{12} \end{bmatrix} = 10^6 \begin{bmatrix} .0170185 & -.0245478 & .00752923 \\ -.0245478 & .0354080 & -.0108603 \\ .00752923 & -.0108603 & .00333103 \end{bmatrix} \begin{bmatrix} y_{10} \\ y_{11} \\ y_{12} \end{bmatrix}$$

Table 4-Torsional stiffness of torque boxes shown in figure 4 for use in computation

($G = 3.947 \times 10^6$ lb/in.$^2$)

| Torque Box | $10^{-6}GJ$, lbs. in.$^2$ |
|---|---|
| I | 31.057743 |
| II | 17.060205 |
| III | 7.219533 |
| IV | 3.744005 |
| V | 45.299802 |
| VI | 29.758063 |
| VII | 13.317379 |
| VIII | 7.871230 |

Table 5-Torsional loads in terms of deflections,
$\{L\} = [\delta]^{-1} \{y\}$.
Stations in figure 4 are referred to by subscripts.

Box I

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = 10^6 \begin{bmatrix} .330315 & -.330310 \\ -.330310 & .330305 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Box II

$$\begin{bmatrix} L_1 \\ L_2 \\ L_4 \\ L_5 \end{bmatrix} = 10^6 \begin{bmatrix} .143618 & -.143629 & -.174723 & .174734 \\ -.143629 & .143640 & .174737 & -.174748 \\ -.174723 & .174737 & .212566 & -.212579 \\ .174734 & -.174748 & -.212579 & .212593 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_4 \\ y_5 \end{bmatrix}$$

Box III

$$\begin{bmatrix} L_4 \\ L_5 \\ L_7 \\ L_8 \end{bmatrix} = 10^6 \begin{bmatrix} .143957 & -.143947 & -.183873 & .183863 \\ -.143947 & .143936 & .183860 & -.183849 \\ -.183873 & .183860 & .234856 & -.234843 \\ .183863 & -.183849 & -.234843 & .234830 \end{bmatrix} \begin{bmatrix} y_4 \\ y_5 \\ y_7 \\ y_8 \end{bmatrix}$$

Box IV

$$\begin{bmatrix} L_7 \\ L_8 \\ L_{10} \\ L_{11} \end{bmatrix} = 10^6 \begin{bmatrix} .095322 & -.095345 & -.129953 & .129976 \\ -.095345 & .095368 & .129985 & -.130008 \\ -.129953 & .129985 & .177167 & -.177198 \\ .129976 & -.130008 & -.177198 & .177230 \end{bmatrix} \begin{bmatrix} y_7 \\ y_8 \\ y_{10} \\ y_{11} \end{bmatrix}$$

Box V

$$\begin{bmatrix} L_2 \\ L_3 \end{bmatrix} = 10^6 \begin{bmatrix} .576995 & -.577026 \\ -.577026 & .577056 \end{bmatrix} \begin{bmatrix} y_2 \\ y_3 \end{bmatrix}$$

Box VI

$$\begin{bmatrix} L_2 \\ L_3 \\ L_5 \\ L_6 \end{bmatrix} = 10^6 \begin{bmatrix} .319500 & -.319518 & -.361665 & .361683 \\ -.319518 & .319536 & .361686 & -.361704 \\ -.361665 & .361686 & .409395 & -.409416 \\ .361683 & -.361704 & -.409416 & .409436 \end{bmatrix} \begin{bmatrix} y_2 \\ y_3 \\ y_5 \\ y_6 \end{bmatrix}$$

Box VII

$$\begin{bmatrix} L_5 \\ L_6 \\ L_8 \\ L_9 \end{bmatrix} = 10^6 \begin{bmatrix} .160351 & -.160344 & -.184729 & .184722 \\ -.160344 & .160338 & .184721 & -.184714 \\ -.184729 & .184721 & .212813 & -.212805 \\ .184722 & -.184714 & -.212805 & .212798 \end{bmatrix} \begin{bmatrix} y_5 \\ y_6 \\ y_8 \\ y_9 \end{bmatrix}$$

Box VIII

$$\begin{bmatrix} L_8 \\ L_9 \\ L_{11} \\ L_{12} \end{bmatrix} = 10^6 \begin{bmatrix} .112283 & -.112273 & -.131573 & .131562 \\ -.112273 & .112262 & .131560 & -.131550 \\ -.131573 & .131560 & .154176 & -.154164 \\ .131562 & -.131550 & -.154164 & .154151 \end{bmatrix} \begin{bmatrix} y_8 \\ y_9 \\ y_{11} \\ y_{12} \end{bmatrix}$$

Table 6 - Load Matrix. $[\Delta]$ in $\{P_1, P_2, \ldots P_{12}\} = 10^6 [\Delta] \{y_1, y_2, \ldots y_{12}\}$

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.351983 | -0.246342 | 0.019089 | -0.179168 | 0.081563 | 0 | 0.028077 | 0 | 0 | -0.002752 | 0 | |
| -.246342 | 1.163532 | -.139261 | .081564 | -.484048 | .047042 | 0 | .096807 | 0 | 0 | -.012048 | 0 |
| .019089 | -.139261 | .334386 | 0 | .047042 | -.218501 | 0 | 0 | .055335 | 0 | 0 | -.008187 |
| -.179168 | .081564 | 0 | .274751 | -.184506 | .015716 | -.103659 | .053635 | 0 | .008565 | 0 | 0 |
| .081563 | -.484048 | .047042 | -.184506 | .603142 | -.101753 | .053634 | -.235507 | .027450 | 0 | .030963 | 0 |
| 0 | .047042 | -.218501 | .015716 | -.101753 | .283655 | 0 | .027450 | -.137092 | 0 | 0 | .025739 |
| .028077 | 0 | 0 | -.103659 | .053634 | 0 | .168049 | -.141266 | .012214 | -.061534 | .050259 | 0 |
| 0 | .096807 | 0 | .053635 | -.235507 | .027450 | -.141266 | .337777 | -.069537 | .050262 | -.112736 | .022731 |
| 0 | 0 | .055335 | 0 | .027450 | -.137092 | .012214 | -.069537 | .157354 | 0 | .022730 | -.056915 |
| -.002752 | 0 | 0 | .008565 | 0 | 0 | -.061535 | .050262 | 0 | .090430 | -.093066 | .007529 |
| 0 | -.012048 | 0 | 0 | .030963 | 0 | .050259 | -.112736 | .022730 | -.093066 | .147708 | -.037496 |
| 0 | 0 | -.008187 | 0 | 0 | .025739 | 0 | .022731 | -.056915 | .007529 | -.037496 | .044892 |

Table 7 - Influence coefficient matrix, $[\Delta]^{-1}$ in $\{y_1, y_2, \ldots y_{12}\} = 10^{-6} [\Delta]^{-1} \{P_1, P_2, \ldots P_{12}\}$ (in./lb.)

| Unit load at station no. | Deflection at station no. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | 10.6606 | 5.3839 | 4.5012 | 18.3460 | 14.1561 | 11.2895 | 25.2424 | 22.4937 | 19.2416 | 32.0979 | 30.2217 | 27.2128 |
| 2 | 5.3839 | 5.1713 | 5.6365 | 12.5056 | 12.6225 | 12.5133 | 19.8021 | 19.8120 | 19.7259 | 26.7618 | 26.7621 | 26.6955 |
| 3 | 4.5012 | 5.6365 | 15.3096 | 13.3441 | 16.1792 | 26.5152 | 24.2199 | 27.1809 | 35.7383 | 35.2185 | 37.5921 | 44.6288 |
| 4 | 18.3460 | 12.5056 | 13.3441 | 49.2445 | 40.3413 | 35.3353 | 77.9417 | 71.0377 | 63.4080 | 104.3948 | 99.7329 | 92.3884 |
| 5 | 14.1561 | 12.6225 | 16.1792 | 40.3413 | 40.9265 | 40.7325 | 70.5397 | 70.7877 | 69.7134 | 99.1668 | 99.2000 | 98.3634 |
| 6 | 11.2895 | 12.5133 | 26.5152 | 35.3353 | 40.7325 | 61.4178 | 67.2523 | 73.7071 | 93.3447 | 100.6213 | 105.9193 | 122.2389 |
| 7 | 25.2424 | 19.8021 | 24.2199 | 77.9417 | 70.5397 | 67.2523 | 155.2519 | 142.1422 | 130.0021 | 224.2203 | 214.6217 | 200.3632 |
| 8 | 22.4937 | 19.8120 | 27.1809 | 71.0377 | 70.7877 | 73.7071 | 142.1422 | 143.3462 | 140.0401 | 215.3920 | 215.7267 | 211.7227 |
| 9 | 19.2416 | 19.7259 | 35.7383 | 63.4080 | 69.7134 | 93.3447 | 130.0021 | 140.0401 | 172.9873 | 207.5273 | 216.6172 | 247.5327 |
| 10 | 32.0979 | 26.7618 | 35.2185 | 104.3948 | 99.1668 | 100.6213 | 224.2203 | 215.3920 | 207.5274 | 384.2291 | 367.2525 | 345.0853 |
| 11 | 30.2217 | 26.7621 | 37.5921 | 99.7329 | 99.2000 | 105.9193 | 214.6217 | 215.7267 | 216.6173 | 367.2525 | 368.6910 | 357.8844 |
| 12 | 27.2128 | 26.6955 | 44.6288 | 92.3884 | 98.3634 | 122.2389 | 200.3632 | 211.7227 | 247.5327 | 345.0853 | 357.8844 | 408.0012 |

Table 8 - Measured influence coefficients.

| One pound load at station no. | Deflection (micro-inches) at station no. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | 24.0 | 7.9 | 8.2 | - | - | - | - | 29.3 | - | 41. | - | 33. |
| 2 | 7.2 | 10.2 | 11.4 | - | - | - | - | 29.0 | - | 37. | - | 43. |
| 3 | 5.6 | 8.0 | 41. | - | - | - | - | 52.2 | - | 63. | - | 104. |
| 4 | - | - | - | - | - | - | - | - | - | - | - | - |
| 5 | - | - | - | - | - | - | - | - | - | - | - | - |
| 6 | - | - | - | - | - | - | - | - | - | - | - | - |
| 7 | - | - | - | - | - | - | - | - | - | - | - | - |
| 8 | 22.8 | 26. | 57.3 | - | - | - | - | 187. | - | 249. | - | 288. |
| 9 | - | - | - | - | - | - | - | - | - | - | - | - |
| 10 | 35. | 32. | 69. | - | - | - | - | 246. | - | 445. | - | 410. |
| 11 | - | - | - | - | - | - | - | 262. | - | - | - | - |
| 12 | 26. | 33. | 97. | - | - | - | - | 277. | - | 399. | - | 680 |

Table 9 - Influence coefficient matrix setting the torsional stiffness of cells I and V, figure 4 to zero, setting the torsional stiffness of cells II and VI to 1/10 the values used in table 6, and setting the torsional stiffness for cells III, VII, IV, and VIII to the values used in table 6.

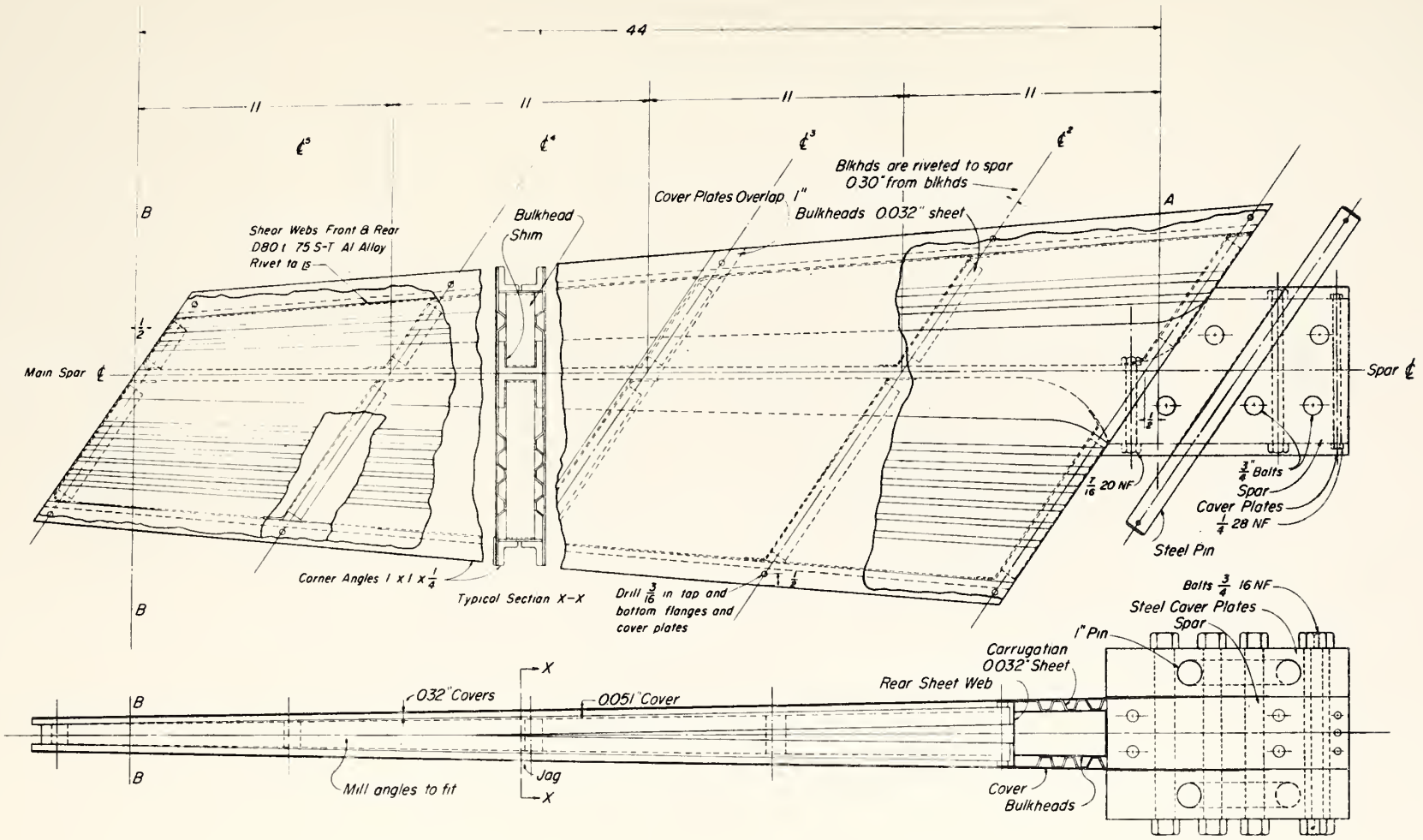
Matrix $[\Delta']$ in $\{P_1, P_2, \ldots P_{12}\} = 10^6 [\Delta'] \{y_1, y_2, y_3, \ldots y_{12}\}$

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.162562 | -0.056918 | 0.019089 | -0.105765 | 0.008156 | 0 | 0.028077 | 0 | 0 | -0.002752 | 0 | 0 |
| -0.056918 | 0.869803 | -0.034953 | 0.008156 | -0.368300 | 0.004704 | 0 | 0.096807 | 0 | 0 | -0.012049 | 0 |
| 0.019089 | -0.034953 | 0.230072 | 0 | 0.004704 | -0.176161 | 0 | 0 | 0.055335 | 0 | 0 | -0.008187 |
| -0.105765 | 0.008156 | 0 | 0.185451 | -0.095200 | 0.015716 | -0.103659 | 0.053635 | 0 | 0.008565 | 0 | 0 |
| 0.008156 | -0.368300 | 0.004704 | -0.095200 | 0.465908 | -0.053828 | 0.053634 | -0.235507 | 0.027451 | 0 | 0.030963 | 0 |
| 0 | 0.004704 | -0.176161 | 0.015716 | -0.053828 | 0.235727 | 0 | 0.027450 | -0.137092 | 0 | 0 | 0.025739 |
| 0.028077 | 0 | 0 | -0.103659 | 0.053634 | 0 | 0.168049 | -0.141266 | 0.012214 | -0.061535 | 0.050259 | 0 |
| 0 | 0.096807 | 0 | 0.053635 | -0.235507 | 0.027450 | -0.141266 | 0.337777 | -0.069537 | 0.050262 | -0.122736 | 0.022731 |
| 0 | 0 | 0.055335 | 0 | 0.027451 | -0.137092 | 0.012214 | -0.069537 | 0.157354 | 0 | 0.022730 | -0.056915 |
| -0.002752 | 0 | 0 | 0.008565 | 0 | 0 | -0.061535 | 0.050262 | 0 | 0.090430 | -0.093066 | 0.007529 |
| 0 | -0.012049 | 0 | 0 | 0.030963 | 0 | 0.050259 | -0.122736 | 0.022730 | -0.093066 | 0.147708 | -0.037496 |
| 0 | 0 | -0.008187 | 0 | 0 | 0.025739 | 0 | 0.022731 | -0.056915 | 0.007529 | -0.037496 | 0.044892 |

Table 10 - Influence coefficient matrix setting the torsional stiffness of cells I and V, figure 4 to zero, setting the torsional stiffness of cells II and VI to 1/10 the values used in table 7, and setting the torsional stiffness for cells III, VII, IV, and VIII to the values used in table 7.

Metrix $[\Delta']^{-1}$ in $\{y_1, y_2, \ldots y_{12}\} = 10^{-6} [\Delta']^{-1} \{P_1, P_2, P_3, \ldots P_{12}\}$ (in./lb.)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 32.3870 | 5.6968 | 18.4034 | 46.4486 | 15.8506 | 27.8319 | 46.8325 | 22.1809 | 23.0065 | 43.9368 | 25.9306 | 13.5085 |
| 5.6969 | 5.2543 | 6.3593 | 12.8046 | 13.2057 | 13.5113 | 20.4689 | 20.7578 | 20.9804 | 27.8260 | 28.0084 | 28.2292 |
| 18.4034 | 6.3593 | 59.5753 | 18.5421 | 22.2583 | 100.9298 | 6.3145 | 42.9827 | 119.4975 | 37.6281 | 65.0147 | 130.7273 |
| 46.4486 | 12.8046 | 18.5421 | 90.4739 | 41.8531 | 21.9860 | 110.2230 | 69.7686 | 0.2078 | 121.8005 | 92.6208 | 31.0947 |
| 15.8506 | 13.2057 | 22.2583 | 41.8531 | 45.0623 | 49.4815 | 74.8933 | 77.7194 | 80.4600 | 106.7333 | 108.5368 | 111.1004 |
| 27.8319 | 13.5113 | 100.9298 | 21.9860 | 49.4815 | 189.6538 | 32.5837 | 98.2343 | 238.0092 | 100.6765 | 149.9832 | 270.0712 |
| 46.8325 | 20.4689 | 6.3144 | 110.2230 | 74.8933 | 32.5836 | 183.4060 | 147.0156 | 92.4183 | 243.9635 | 217.3616 | 165.8350 |
| 22.1809 | 20.7578 | 42.9827 | 69.7687 | 77.7194 | 98.2342 | 147.0156 | 156.2345 | 168.9764 | 228.0028 | 234.2255 | 244.0384 |
| 23.0064 | 20.9804 | 119.4975 | 0.2079 | 80.4600 | 238.0092 | 92.4184 | 168.9765 | 336.6417 | 209.3623 | 267.8757 | 415.2037 |
| 43.9368 | 27.8260 | 37.6280 | 121.8006 | 106.7333 | 100.6764 | 243.9635 | 228.0028 | 209.3622 | 405.2323 | 383.1642 | 351.2017 |
| 25.9306 | 28.0084 | 65.0147 | 92.6209 | 108.5368 | 149.9831 | 217.3617 | 234.2255 | 267.8756 | 383.1642 | 396.3146 | 413.6454 |
| 13.5084 | 28.2292 | 130.7273 | 31.0948 | 111.1004 | 270.0711 | 165.8351 | 244.0384 | 415.2036 | 351.2018 | 413.6455 | 508.7067 |




ASSEMBLY DRAWING
SWEEPBACK = 35° AT MAIN SPAR
ALL DIMENSIONS IN INCHES
RIVETS NOT SHOWN
TOLERANCES TO AIRCRAFT MANUFACTURERS
PRACTICE EXCEPT AS NOTED

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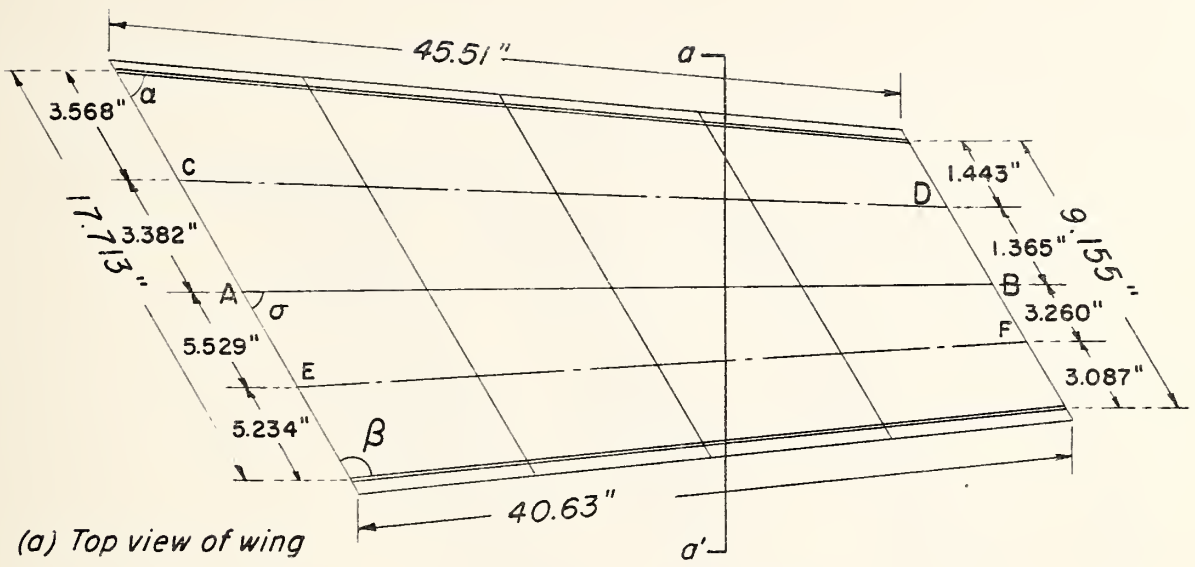
Fig. 1. Test Wing.




(a) Top view of wing

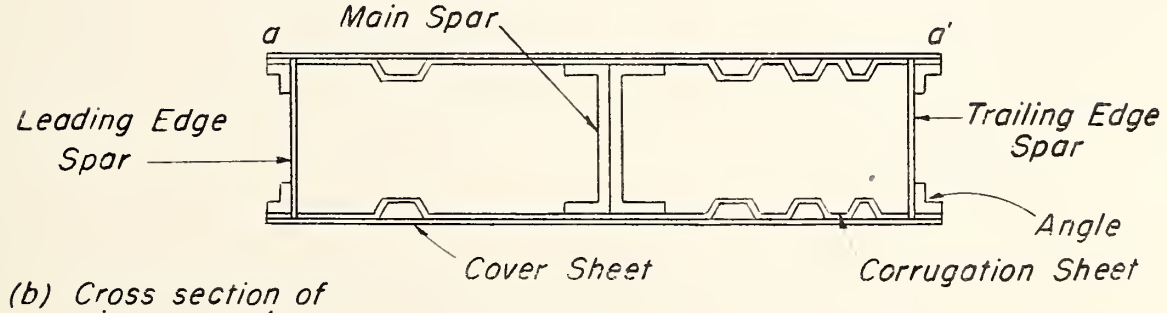
$\alpha$ = 50.723°

$\beta$ = 119.887°

$\sigma$ = 55.0°

A B = 43.0 in.

C D, E F = lines midway between spar center lines




(b) Cross section of wing at a–a'.

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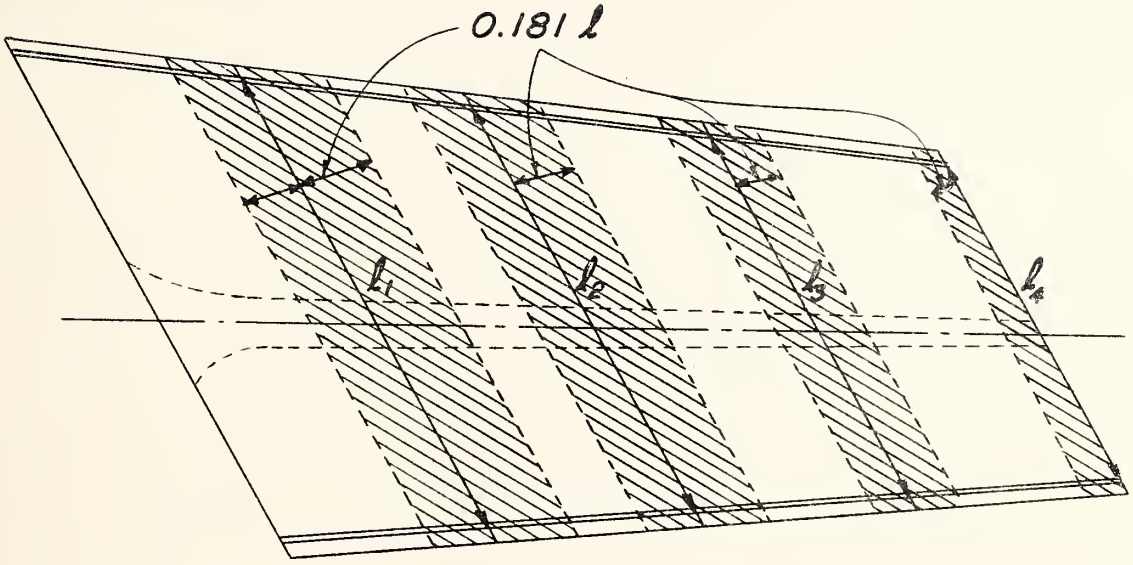
Fig. 2 Geometry of Wing.




Chordwise Beams

| Rib | ℓ | 0.181 ℓ |
|---|---|---|
| 1 | 15.638" | 2.830" |
| 2 | 13.449" | 2.434" |
| 3 | 11.266" | 2.039" |
| 4 | 9.155 | 1.657" |

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Fig. 3 Selection of chordwise beams.

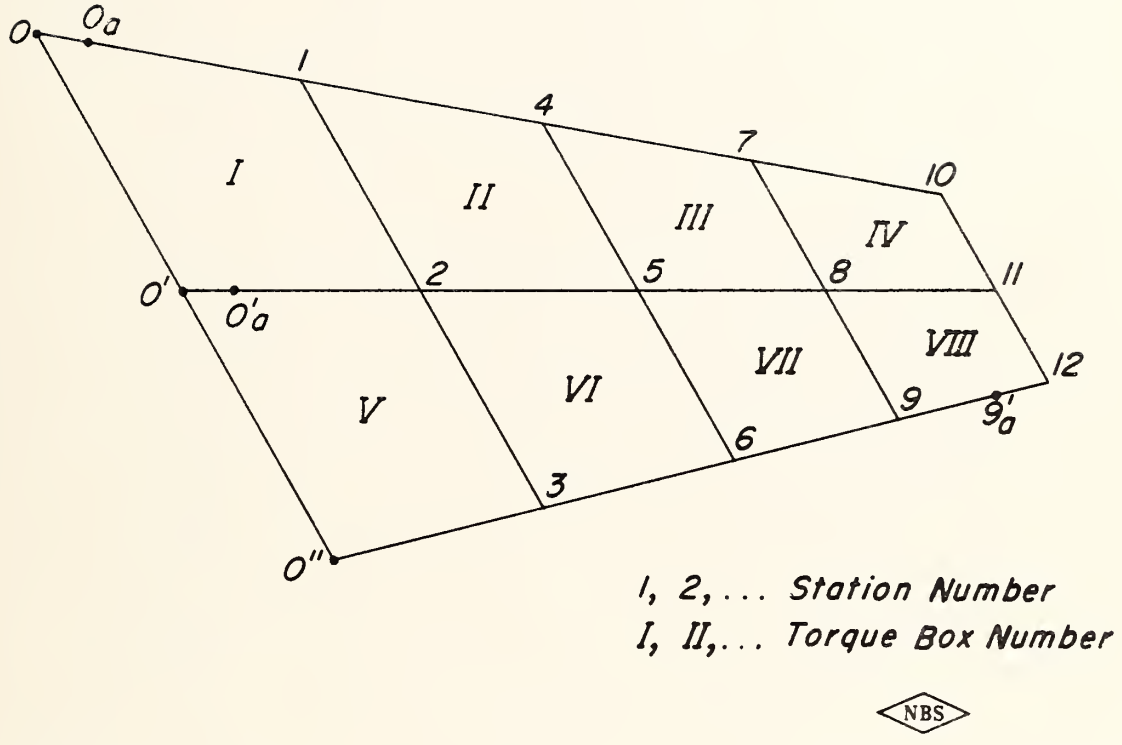


Fig. 4. Numbering of stations on wing. Designations for wing torque boxes.

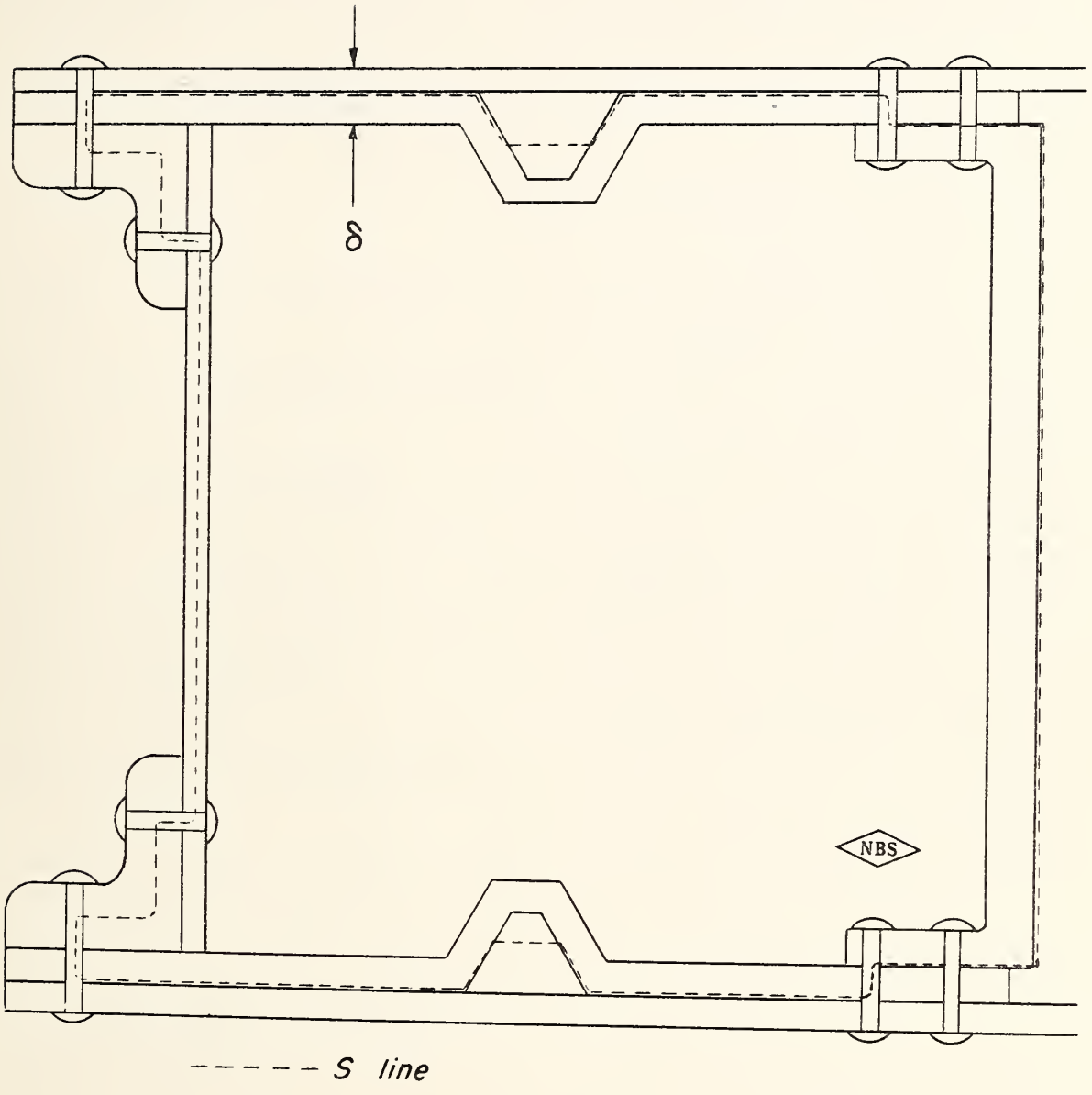


Fig. 5 Cross-section for GJ computation for a leading edge torque box showing location of line S.

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